Where is the Shot Noise of a Quantum Point Contact?

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Reznikov *et al.* (Phys. Rev. Lett. **75**, 3340 (1995)) have presented definitive observations of nonequilibrium noise in a quantum point contact. Especially puzzling is the "anomalous" peak structure of the excess noise measured at constant current; to date it remains unexplained. We show that their experiment directly reveals the deep link between conservation principles in the electron gas and its low-dimensional, mesoscopic behavior. Key to that connection are gauge invariance and the compressibility sum rule. These are central not only to the experiment of Reznikov *et al.* but to the very nature of all mesoscopic transport.

PACS numbers: 71.10.Ca, 72.10.-d, 72.70.+m, 73.23.-b

Microscopic conservation laws are the *sine qua non* of transport and fluctuation physics. Their dominance is even more apparent in the passage to sub-micrometer electronics. In the following, we bring to light the direct governing role of conservation in low-dimensional mesoscopic conduction, with immediate experimental consequences. Indeed, these have already been observed [1].

The primary global statements of conservation are the electron-gas sum rules [2]. Their cardinal significance as conserving relations is that they must be satisfied *automatically* by credible models of mesoscopic transport [3].

This paper details the particular and striking interplay of the compressibility sum rule and dynamical electron relaxation. That interplay dominates the form of the carrier fluctuations (noise) of a driven quasi-one-dimensional quantum point contact, so completely as to dictate the shape of its nonequilibrium noise spectrum. Our results have far-reaching implications for understanding transport in mesoscopic electron devices.

Some years ago Reznikov, Heiblum, Shtrikman, and Mahalu [1] performed a landmark experiment on nonequilibrium noise in a quantum point contact (QPC). The electron density was freely adjusted via a gate voltage V_g, sweeping the carriers from their low-density classical regime up to high density and degeneracy. At fixed values of source-drain voltage V across the QPC, a regular sequence of peaks appeared in the noise power spectrum as the channel's carrier density was systematically increased. Analogous features were also seen by Kumar *et al.* [4]. The behavior at fixed V is predicted by the noninteracting one-electron picture of coherent ballistic conduction, first applied to QPC shot noise by Khlus [5]. It has since been refined and redefined by Landauer, Büttiker, Imry, and many others; for an authoritative review see Ref. 6 and citations in it.

A major innovation of Reznikov *et al.* was to measure the nonequilibrium noise for fixed levels of source-drain *current* I through the QPC, as well as their standard measurements at fixed V. A surprise ensued. Far from the anticipated strictly monotonic falloff of spectral strength [6], the data exhibited a series of pronounced noise maxima at the lowest subband energy threshold.

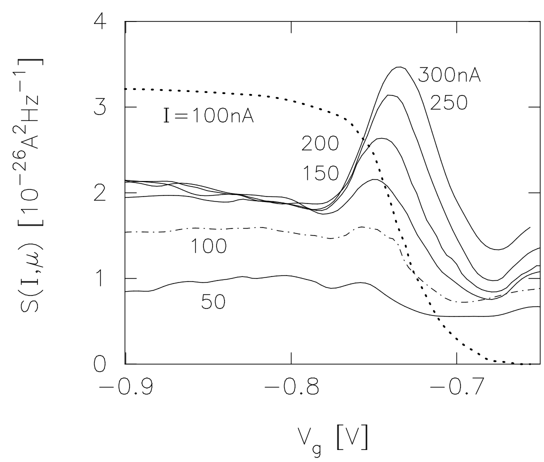

FIG. 1: Nonequilibrium current noise in a quantum point contact at 1.5K, measured by Reznikov *et al.* [1] as a function of gate bias, at fixed source-drain current. Dotted line: the most widely adopted theoretical noise model [6] typically produces a strictly monotonic noise signal at the first subband energy threshold. That model totally fails to predict the very strong noise peaks actually observed at threshold.

Figure 1 shows the constant-current data of Ref. 1. At higher source-drain currents, a robust peak structure emerges in the noise as a function of gate bias, precisely at the location of the first subband threshold. At $I =$ 100nA, we reproduce the conventionally predicted shot-noise curve [1, 6].

Reznikov *et al.* remark that "the peaks near $\mathcal{T}_1 = 1/2$ [*ie* at threshold] are not expected" [1]. This is an understatement, for the observed excess-noise maxima

contradict the accepted theoretical predictions, based on shot noise. Something quite uncommon and apparently inexplicable is going on.

In the nine years since publication of that experiment, its astonishing outcome has not been revisited. Yet the anomalous signatures are first-hand evidence that a deep knowledge gap exists in our physical understanding of mesoscopics. This has ramifications not just for quantum point contacts, but for all low-dimensional devices.

We now analyze the Reznikov *et al.* experiment [1]. A straightforward kinetic approach explains both constant-voltage and constant-current results, while automatically securing conservation. We discuss why the nonequilibrium noise of a quantum point contact cannot be some kind of shot noise, as commonly believed: it is a hot-electron fluctuation effect unique to quasi-one-dimensional mesoscopic conductors. Shot noise – if present at all – is unimportant here.

Our treatment starts with orthodox quantum transport in metals [7, 8, 9]. A uniform one-dimensional ballistic wire is intimately contacted to large source and drain reservoirs. The reservoir leads, permanently neutral and equilibrated, pin the local chemical potential at each interface with the wire. At the interfaces, the perturbed carrier distribution goes smoothly to its equilibrium form in the leads, set by the local chemical potential.

A generator forces an electron current into the wire at the source and removes it through the drain. In open-system conduction, the active injection and extraction of external current is *necessary and sufficient* to ensure charge conservation not only microscopically, but globally [10]. This is a nontrivial and mandatory constraint on any microscopic account of noise and transport.

The external current evokes a resistivity-dipole field E within the uniform wire as the carrier density at both interfaces adjusts to the disturbance. The standard kinetic equation for the electron distribution $f_k(t)$ is

$$\frac{\partial f_k(t)}{\partial t} + \frac{eE}{\hbar}\frac{\partial f_k(t)}{\partial k} = -\frac{1}{\tau_{\rm in}(\varepsilon_k)}\left(f_k(t) - \frac{\langle \tau_{\rm in}^{-1} f(t)\rangle}{\langle \tau_{\rm in}^{-1} f^{\rm eq}(\mu, k_{\rm B}T)\rangle} f_k^{\rm eq}(\mu, k_{\rm B}T)\right) - \frac{1}{\tau_{\rm el}(\varepsilon_k)}\frac{f_k(t) - f_{-k}(t)}{2}. \tag{1}$$

The inelastic and elastic relaxation rates, $1/\tau_{\rm in}$ and $1/\tau_{\rm el}$ respectively, parametrize the collision term. They may depend on subband electron energy ε_k. Expectations $\langle ... \rangle$ trace over wavevector space and spin; for example, $\langle \tau_{\rm in}^{-1} f(t)\rangle = 2\int \tau_{\rm in}(\varepsilon_k)^{-1} f_k(t) dk/2\pi$. The equilibrium function $f^{\rm eq}(\mu, k_{\rm B}T)$ has the usual Fermi-Dirac form, dependent on chemical potential μ and thermal energy $k_{\rm B}T$.

Crucially, the collisional structures on the right of Eq. (1) secure charge and number conservation. These essential properties are inherited by the dynamical mean-square number fluctuation $\Delta f_{\rm k}(t)$ in the structure, determining all the correlation functions of physical interest [8, 11]. The equation for $\Delta f_{\rm k}(t)$ is systematically generated by varying both sides of Eq. (1). Microscopic conservation is built in. *So are the sum rules* [2, 3].

In the low-frequency limit the noise spectral density in a QPC is duly obtained. For a device of operational length L, subject to current I, it has the functional form

$$S^{\rm xs}(I, \mu) = \int_0^\infty dt \int_{-L/2}^{L/2} dx \int_{-L/2}^{L/2} dx' \Big(C_{JJ}(x - x'; I, \mu, t) - C_{JJ}(x - x'; I = 0^+, \mu, t) \Big); \tag{2}$$

we remove the equilibrium Johnson-Nyquist noise floor ($I \to 0$). The nonequilibrium current-current correlator $C_{JJ}(x - x'; I, \mu, t)$ is computed from the Green function for the spatially dependent version of Eq. (1). For further details of our QPC model see Ref. 11.

Equation (1) and its variations are not restricted to weak applied currents. C_{JJ} is derived within a fully nonequilibrium description, not only conserving but equally valid at strong fields [8] as at weak. The ability to do this, in a strictly conserving way, is an absolute prerequisite for analyzing the experiment of Ref. 1, whose conditions take a QPC far out of the weak-field, highly degenerate limit addressed by Khlus [5] and others [6].

For collision times $\tau_{\rm el}$ and $\tau_{\rm in}$ independent of subband energy ε_k, the nonequilibrium problem is exactly solvable [11]. We now focus on a physical model for the behavior of these times in a strongly nonequilibrium environment.

Let us start with elastic scattering. Since the quantum point contact is impurity-free, its elastic mean free path (MFP) is matched to L, the operational length of

the ballistic core. The elastic scattering rate will not be sensitive to the driving field. The characteristic velocity of the carriers is $v_{\rm av} \equiv \langle |v| f^{\rm eq} \rangle / n$ at the QPC electron density $n = \langle f^{\rm eq} \rangle$. Thus the elastic time is

$$\tau_{\rm el} = L/v_{\rm av}. \tag{3}$$

In the pinchoff limit, far below the subband threshold ε_1, the density $n \sim e^{-(\varepsilon_1 - \mu)/k_{\rm B}T}$ vanishes. The carriers are classical; $v_{\rm av}$ is thermal. For $\mu - \varepsilon_1 \gg k_{\rm B}T$ (degenerate limit) $v_{\rm av} \to v_{\rm F}$; it is the Fermi velocity in the subband.

The behavior of $\tau_{\rm el}$ reflects the constancy of the elastic MFP. The inelastic MFP, however, will decrease substantially with increasing source-drain voltage. The field-excited electrons will shed excess energy by emitting many more phonons. We model this inelastic loss via

$$\tau_{\rm in}(V,\mu) = \tau_{\rm el} \left[\frac{\varepsilon_{\rm av}}{\alpha e V} \tanh\left(\frac{\alpha e V}{\varepsilon_{\rm av}} \right) \right]^{\beta}. \tag{4}$$

The driving voltage is $V = EL$ while $\varepsilon_{\rm av} = \frac{1}{2} m^* v_{\rm av}^2$ is the characteristic energy of the subband population; it is thermal near pinchoff and Fermi at high filling. The ratio $eV/\varepsilon_{\rm av}$ determines the inelasticity. At high degeneracy, $\varepsilon_{\rm av} = \mu - \varepsilon_1 \gg k_B T$ and Pauli exclusion inhibits phonon emission. The parameters α and β are set once (to $\alpha = 0.52$ and $\beta = 0.6$) to match the peaks at 250 and 300nA in Fig. 1. These values then determine all our results.

In the low-field limit, $\tau_{\rm in}(V,\mu) \to \tau_{\rm el}$. This is the condition for ideal ballistic transport in an open contact, when all dissipation is in the leads. It yields Landauer's quantized conductance steps across the subband thresholds [9]. As field-induced phonon emission sets in, the inelastic MFP rapidly shrinks below L. Eq. (4) encodes this nonideal behavior.

In our parametrized inelastic model, Eq. (2) for the excess noise in the first QPC subband becomes [11]

$$S^{\rm xs}(I,\mu) = \frac{2e^2 I^2}{m^* L^2} \frac{1}{G(I,\mu)} \times \frac{\kappa}{\kappa_{\rm cl}} \left(\tau_{\rm in}^2 + 2\frac{\tau_{\rm el}\tau_{\rm in}^2}{\tau_{\rm el}+\tau_{\rm in}} - \frac{\tau_{\rm el}^2\tau_{\rm in}^2}{(\tau_{\rm el}+\tau_{\rm in})^2} \right). \tag{5}$$

The conductance $G(I,\mu) = I/V$ is

$$G(I,\mu) \equiv \frac{2e^2}{h} \mathcal{T}_1(I,\mu) = \frac{2e^2}{h} \left(\frac{v_{\rm F}}{v_{\rm av}} \right) \frac{2\tau_{\rm in}}{\tau_{\rm el}+\tau_{\rm in}}. \tag{6}$$

At fixed current this determines the corresponding voltage. The relation is nonlinear owing to the implicit V-dependence through $\tau_{\rm in}$ from Eq. (4). Second, *as required by its sum rule* [3], the compressibility $\kappa = n^{-1} \partial \ln n / \partial \mu$ appears (in ratio with its classical value $\kappa_{\rm cl} = 1/n k_{\rm B} T$).

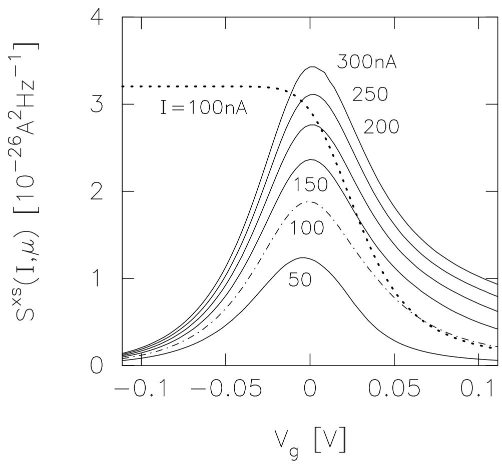

FIG. 2: Excess hot-electron noise at 1.5K in a QPC at its first subband threshold, computed with our strictly conserving Eq. (5), as a function of gate voltage (relative to the first threshold), at fixed levels of source-drain current. There is close quantitative affinity of our peaks to the experimentally observed first-threshold maxima in Fig. 1 [1]. Dotted line: corresponding prediction at $I = 100$nA from a widely adopted theory of QPC noise [6], using our transmissivity $\mathcal{T}_1$ (Eq. (6)) as that model's phenomenologically required input. Compare it with the kinetic result at 100nA (chain line).

The last factor on the right of Eq. (5) is highly sensitive to the ratio $\tau_{\rm in}/\tau_{\rm el}$ out of equilibrium [11].

In Fig. 2 we display the result of our conserving computation, implementing the physics of Eqs. (3)–(6). Most dramatic are the peak structures at the threshold $\mu = \varepsilon_1$ as the chemical potential takes n in the channel from low to high values. The quantitative fit to the as-measured peaks of Fig. 1 is noteworthy. The conventionally predicted shot-noise analog [6] is entirely wide of the mark.

What are the peaks? The peaks are a snapshot of the carriers' transition from classical to quantum behavior. They are generated by strongly competing trends within $S^{\rm xs}(I,\mu)$: the compressibility κ, and the combination of collision times in the numerator with $G(I,\mu)$ in the denominator. Consider the limiting cases.

(i) *Degenerate limit.* Then $\kappa/\kappa_{\rm cl} \to k_{\rm B}T/2(\mu - \varepsilon_1) \ll 1$ even as the collision-time factor reaches its ideal maximum of $1.75\tau_{\rm el}^2$ and G plateaus out at $2e^2/h$. We have

$$S^{\rm xs}(I,\mu) \propto (\kappa/\kappa_{\rm cl})(\tau_{\rm el}/L)^2 \sim n^{-4}. \tag{7}$$

(ii) *Pinchoff limit.* At low densities, the carriers are classical. The compressibility ratio $\kappa/\kappa_{\rm cl}$ goes to a maximum of unity. At low n, though, carriers are highly energized. Eq. (4) tells us that

$$V = I/G \propto I/(n\tau_{\rm in}(V,\mu)) \to I/(nV^{-\beta}). \tag{8}$$

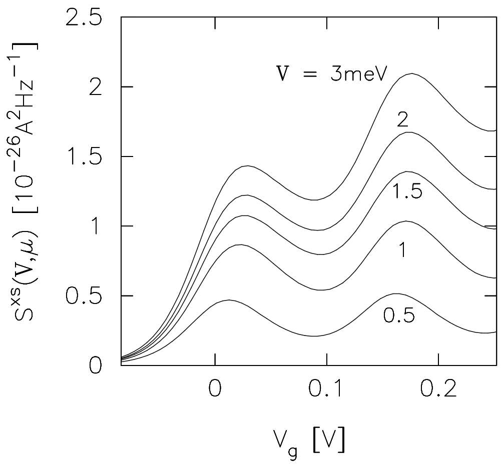

FIG. 3: Excess hot-electron noise in a QPC at 1.5K for its first two subbands, at constant applied voltage V, as a function of gate voltage (referred to 1st threshold). This corresponds to Fig. 2 of Ref. 1. The quasi-linear dispersion of the peaks with V shows that such dispersion is not unique to shot noise.

That is, $eV(I,\mu) \sim n^{-1/(1-\beta)} \gg \varepsilon_{\rm av}$. The direct dependence of $S^{\rm xs}$ on the collision times, along with $G \propto n\tau_{\rm in}$ in the denominator, then yields the overall asymptote

$$S^{\rm xs}(I,\mu) \propto \quad (\kappa/\kappa_{\rm cl})\tau_{\rm in}(V,\mu)/n \\ \sim \quad (n^{-1/(1-\beta)})^{-\beta}/n = n^{(2\beta-1)/(1-\beta)}. \quad (9)$$

In either case Eqs. (7) and (9) lead to vanishing shoulders. In the neighborhood of the threshold, the peaks in $S^{\rm xs}(I,\mu)$ are the outcome of the compressibility, heralding the onset of degeneracy, playing against the dynamics of nonequilibrium dissipative scattering in the channel.

We present in Fig. 3 the results of our noise model at constant voltage. Comparing it with the corresponding Fig. 2 of Reznikov *et al.* [1], we again see a concordance between observations and the kinetic calculation.

It is important to examine two aspects of the Reznikov *et al.* noise data, in light of what is needed to explain it.

(a) *Linear response is inapplicable.* A quick estimate shows that the experiment lies well beyond the limits of weak-field linear models which perturb a system mildly away from its Fermi surface [6]. In the experimental regime of Fig. 3 the gate voltage sweeps the channel's Fermi energy from larger, safe values (calibrating G against measurements, we see that $\mu \approx$ 3meV covers three subbands), far down to where carriers are not degenerate at all, but classical.

In Fig. 3 the applied voltage goes up to 3meV. Since $eV \gtrsim \mu$, linear response is clearly unjustified. Much more extreme is Fig 2, for constant current. Eq. (8) shows just how rapidly V reaches enormous values at channel densities below threshold.

The severe limitation of weak-field models has not inhibited their wide use in data analysis [1, 4, 6]. We stress that a properly constructed, fully nonequilibrium kinetic approach – one that is strictly conserving – is the only appropriate analytical tool.

(b) *There is no shot noise.* Eq. (2) and its specific form Eq. (5) do not describe shot noise. They describe hot-electron noise, whose thermodynamic origin is different [8]. This is borne out starkly by the peak structures of Fig. 1, in patent contradistinction to shot-noise based predictions [6]. Moreover, near-linearity of the peaks in Fig. 3 with V is not special to shot noise [1]. Such dispersion arises naturally from inelastic dynamics.

Finally, readers will note that the noise plateaux at very low densities in Fig. 1 are not reproduced by those models that enforce full shot noise (whose plateaux are far too big), nor by the collisional model, in its presently simplified form. What is really needed is a fuller theory that can cross from the one-dimensionally confined QPC state to one where the low-density, high-energy electrons are so excited as to break the confinement and fan out, nonuniformly and at high momenta, into the drain.

We have presented a microscopically grounded theory of nonequilibrium mesoscopic noise. It is based on standard and rigidly conservative kinetics. Thus it is well controlled, and the whole range of relevant noise properties of a quantum point contact [1, 4] is well reproduced. For the first time, we explain qualitative and quantitative experimental features that other theories [6] miss entirely.

We have accounted for the prominent and enigmatic noise peaks that have defied explanation until now. The central role of the electronic compressibility has been identified and quantified as an essential physical determinant of mesoscopic fluctuations. That is true not only in QPCs but in all conductive structures at these scales.

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